Dynamic Overbooking Air Transportation Model – Single-Leg Airline Revenue Management & Relative Adjustment to Covid-19

FINAL PROJECT: MATH MODELING

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Abstract

Generally speaking, airline passengers can start booking tickets around 30 days before the plane takes off. However, due to other external factors, some passengers who have booked tickets cannot board the plane, thus causing the phenomenon of empty seats on the plane and wasting resources. To maximize the benefits, airlines adopt an overbooking strategy, but it also brings other risks.

Based on passenger behavior, fluctuation rules of ticket prices before takeoff, and relevant policies of airlines to deal with no-show passengers and other emergencies, this paper establishes a dynamic model for aircraft overbooking decision and puts forward reasonable suggestions for the sales tickets of each day based on the fitted number of tickets and plane price per day.

Considering the COVID-19 epidemic, we further adjust the original dynamic model based on the actual situation and establish a new model for COVID-19 aircraft overselling decisions, aiming to provide more reasonable overselling suggestions for airlines under the influence of COVID-19. Using Analytic Hierarchy Process (AHP) and basic SIR model, we quantify the heightened security and passengers' fear and connect them with passengers' no-show probability. Through numerical verification, the dynamic model can increase the profit of airlines by 11% under normal conditions but not significantly increase the profit of airlines under the influence of COVID-19 as ticket orders are less than actual capacity. Thus smaller planes could be applied to increase the rate of loading and dynamic model works well. Overall, our dynamic model is of great significance to reduce airline losses and give rise to overall profit.

Chapter 1

Introduction

1.1 Background Information

No-show & Denied Boarding Cases

The tactic rule of an airline reservation is that, passengers can book tickets via the ticket office or the airline more than ten days before the departure time of the plane. Due to the relatively long period from the departure time, as well as the uncertainty of passenger behavior under complicated situations, often the airline will sell more tickets than the actual seats to avoid potential loss of benefit, namely overbooking.

In the ticket booking decision, airline companies generally face two risks: the risk of empty seats due to the booked passengers' not showing up and the risk of overbooking due to the overselling method adopted by airline companies, and both of them will bring the loss of benefit. Taking the passenger capacity of flights as the critical point: if the number of passengers who arrive at the airport with reserved seats is less than the flight capacity, there will be seat surplus, which is the risk of empty seats; contrarily, if the number of arrived passengers is more than the flight capacity due to the overselling method, some passengers will be denied to board the plane, thus causing the risk of companies' unnecessary reparations and further damage to reputation. Generally saying, a reasonable overselling method can reduce the loss of seats and assist in boosting total profit with relatively slight influences on corresponding risks, but combining various kinds of uncertainty, it is very difficult to determine the reasonable overselling amount.

Overbooking plays one of the significant rules in revenue management for airline companies, which is always coming along with two major issues, which we have stated previously: No-Show problem (the reserved passengers do not show up, leading to empty seats as a waste of resources), and Denied-Boarding (DB) problem (the number of arrived passengers is larger than airline capacity due to the ticket-overselling method, leading to reparation and credit risk).

Considering real practices, airlines often have no-show passengers, creating some empty seats while some passengers demand tickets on-site (also called Co-Show) but cannot manage the trade since the tickets are literally sold out.

According to the analysis of historical sales and departure data, the No-Show rate of passengers can be predicted, and thus the oversold rate can be determined for air ticket sales. Through possible predictions, we can not only make the most of the seats available on hot-line flights, increasing airline revenues together with efficiency, but also make it possible for other Co-show travelers to fulfill the requirement, which creates a win-win situation. For instance, Lufthansa does an excellent job of overbooking, generating 5% more revenue per year.

Notice that the overselling forecast cannot be accurately generated, the so-called DB (Denied) entry problem occurs. This causes dissatisfaction with passengers and even conflicts between airlines and passengers. In unusual, airline companies take the approach of compensating DB passengers while such compensation is often more than twice the ticket price. In the event of DB, the cost of airline companies rises rapidly, which is also what they are not willing to see.

Therefore, overselling is a double-edged sword. How to solve the contradiction between the No-Show rate and DB has always been an issue of great concern to airlines and academic circles.

By far, the over-booking method of the airlines in China has been static. That is to say, the number of oversells remains the same from the open date until the departure date for a flight according to government regulation. That ignores many factors, such as the fluctuation of ticket prices, the difference between Business Class and Economic Class, and so on. That leads the airline companies far away from the best-selling strategy.

In the process of selling tickets, the airline's reservation system accepts the passengers' requirements for both the booking of a ticket and the cancellation or rescheduling of another flight. Tickets should be booked much faster than the cancellation rate, and at some point, before the plane takes off, it will reach or approach the capacity of the plane, at which point the airline will face the overbooking problem. Airlines can control the volume of tickets booked and will no longer accept requests for tickets when the number of booked tickets exceeds the desired number. However, due to the uncertainty of booking demand, the current rejected demand will no longer appear in the future, and the future cancellation will continue to occur, then there will be empty seats when the plane takes off, resulting in the decline of flight revenue.

Therefore, the overbooking of airline tickets is a dynamic decision-making process. This process depends on the current sales status, future demand distribution, ticket cancellation distribution, and no-show rate at takeoff.

The Overview of Airline Transportation under COVID-19

The outbreak of COVID-19 in Wuhan began in mid-December 2019, and by February 3, 2020, the number of infected people had reached 17,238 (according to the unified national epidemic report data), among which there were 21,558 suspected cases. According to the confirmation rate of around 50%, the infected people should exceed 25,000. In the 7 months of SARS in 2003, the number of infected people in the world was only about 8,100, and the number of infected people in Wuhan was 2-3 times higher than that in the period of SARS in 2003, which was far more infectious and influential than THE SARS virus.

Here are major negative influences that COVID-19 has brought to the airline transportation system in China[1].

- Less available flights and more cancellation:

  On January 25, the China Travel Association issued an announcement that "all group tours of travel agencies across the country will be suspended". As a result, from January 24th to 31st, some or all flights involving Wuhan and Hubei provinces began to be canceled. From February 1, China's foreign airlines began to cancel a large number of flights to China, and from February 1, China's airlines began to cancel a large number of remaining domestic flights. Flight cancellations will have a huge impact on the second half of the Spring Festival travel rush, which runs from Jan 24th to Feb 18th.

- Significant decrease in passenger traffic:

  In the face of widespread flight cancellations that began on 24 January, passenger traffic has been greatly affected, particularly since 1 February. As of February 1, a total of 34.36 million passenger trips had been made during the Spring Festival travel rush, down 17.1 percent year on year, according to the civil Aviation Resources Network. Among them, the passenger transport volume on February 1 was only 470,000, a sharp decrease of 76.4% year on year, and the one-day passenger load factor of all civil aviation was only 43.58%, setting a new low since the Spring Festival travel rush.

- The utilization rate of aircraft of each airline has decreased significantly:

  The Spring Festival travel rush has always been the most critical 40 days for airlines to consolidate the business performance in the first half of the year. Airlines will also try to increase the aircraft utilization rate during the Spring Festival travel rush to the maximum flight hour that the company's flight crew can execute. During the Spring Festival travel rush, the department of Small and medium-sized airlines, taking advantage of the overtime policies of the General Administration and regional administrations, began to increase flights on busy routes during the Spring Festival travel rush. Basically, the target of aircraft utilization required by the Department of small and medium-sized airlines would be more than 11 hours. According to the actual situation of an airline company, from January 10 to 23, the utilization rate of the aircraft reached more than 11 hours, which was reduced to about 9 hours after the shift reduction on January 24, and less than 5 hours after the significant shift reduction on February 1. A sharp decline in utilization would lead to a sharp rise in unit costs, which would increase the variable costs of the remaining flights and lead to negative flight edges.

- Passenger load factor decreased significantly, especially for international flights [2]:

  On the evening of 30 January, WHO announced that the Novel Coronavirus outbreak had been listed as a public health emergency of international concern. Since January 31, Italy, the United States, Australia, New Zealand, Singapore, and other countries have started to cancel flights to China on a large scale, and more than 70 countries have suspended the entry of Chinese citizens or imposed corresponding restrictions.

  Originally, the department of Domestic Aviation relied heavily on travel groups for its international flights. Now, it is prohibited to organize travel groups to receive the documents of the National Tourism Administration with no termination date. Combined with the double impact of international restrictions on Chinese citizens entering the country, international flights carried out by domestic companies have been largely canceled since February 1 and will be completely canceled by the end of February. Less than 50% of the remaining flights of each airline company are in flight. As the movement of people is almost banned across the country, the willingness of passengers to travel is greatly reduced and reaches the freezing point. In addition, the Civil Aviation Administration has made it clear that a full refund of tickets must be made after 24 days, passengers who have booked tickets in advance cancel their trips, and no new passengers have been booked later. As a result, the passenger load factor of the remaining flights will not reach 50%.

1.1.1 Research Purpose & Significance

In this paper, we try to build a dynamic model for airline companies' overbooking decisions. That is, according to the simulated number of tickets booking every day and the expectation of the total number of tickets booking in the future, we use MATLAB as the major tool to simulate the possible permutation and combination of multiple inducts. By comparing all the results we get from different combinations, we can obtain the suggested sales tickets for the day, improve the profit of airlines, and reduce the risk brought by overbooking decisions.

Meanwhile, we are considering the specialty of the current situation according to the given four external factors brought by Covid-19. Through the establishment of a new simple SIR epidemic model, considering the above factors for the effects of parameters and variables, to further improve the model, and under the influence of the outbreak of the new champions league for more than an outbreak of airlines in different stages to take overbooked policy recommendations. This provides a reference for airlines severely affected by Covid-19 to adjust their decisions in real-time to reduce losses or increase profits. It is of certain reference value to the airline ticket sales and the decision of the relevant government when encountering similar epidemic impacts or other irresistible factors in the future.

1.2 PROBLEM RESTATEMENT

Problem A: Airline Overbooking

Historically, airlines know that only a certain percentage of passengers who have made reservations on a particular flight will actually take that flight. Especially, now due to the Covid-19, a lot of passengers would like to get the tickets as earlier as possible in order to come back to hometown. However, they need canceled the fight due to some safety concern, physical condition or some other isolation policies. Consequently, most airlines overbook-that is, they take more reservations than the capacity of the aircraft. Occasionally, more passengers will want to take a flight than the capacity of the plane leading to one or more passengers being bumped and thus unable to take the flight for which they had reservations.

Airlines deal with bumped passengers in various ways. Some are given nothing, some are booked on later flights on other airlines, and some are given some kind of cash or airline ticket incentive. Consider the overbooking issue in light of the current situation:

- Less flights by airlines from point A to point B
- Heightened security at and around airports
- Passengers' fear
- Loss of billions of dollars in revenue by airlines to date

Build a mathematical model that examines the effects that different overbooking schemes have on the revenue received by an airline company in order to find an optimal overbooking strategy, i.e., the number of people by which an airline should overbook a particular flight so that the company's revenue is maximized. Insure that your model reflects the issues above, and consider alternatives for handling ”bumped” passengers.

1.2.1 Problem Analysis

Fundamentally, we build a dynamic overbooking model to determine the overbooking amount on each day in the whole pre-sale period to maximize the revenue. Considering the price fluctuation, we apply Monte Carlo simulation to predict prices on each day using historical statistical data. Further, by the basic demand principle in economy, we construct the relation between prices and actual selling tickets amount by a fraction function for both economy and business classes.

In the COVID-19 setting, we adjust our model in the following detailed ways:

For the first criteria, when there are flights from point A to point B, since we only consider a single flight at one time between two destinations in our model, less flights could be considered as less capacity of a single flight on average given the total number of flights unchanged. Thus we may adjust c and c_s to be smaller.

For the second and third ones, heightened security and passengers' fear would make people with ordered tickets more reluctant to be on board in time. That implies that the probability for passengers to be no-show in both economy and business classes would be larger than before in our context. Thus here we introduce AHP model to quantify the relation between no-show probability and passengers' fear, heightened security factor.

When huge deficit is expected to happen, airline companies would tend to sell more tickets in the pre-sale period with the possibility that more bumped passengers would occur to increase the loading rate. That means the maximum overbooking rate should be larger in our model to allow more overbooking.

1.2.2 Assumptions & Notations

- Whether or not passengers arrive on time is independent of each other (this applies to businessmen and tourists acting alone);Each passenger decides independently to have travel demands, order the ticket and board a plane in time. In other words, traveling in groups is not considered in this model.

- The revenue considered in our model is for a single flight from point A to point B and the total revenue for the whole company is just the sum of all of such flights.
- Air fares fluctuate regularly over time, and the closer the departure time is, the higher the fare is.
- For each flight, the company sells more tickets than its actual capacity of loading and is willing to take the risk of passengers' no-show and denied boarding conditions.
- Each flight has two classes: Business Class (BC) and Economy Class (EC): The BC passengers have the willing to pay higher but has lower probability to show up; the EC passengers pay lower prices and have higher possibility to show up. BC passengers can get more reparations than EC passengers when not show up.
- When economy class is full, the extra passengers will automatically accept the compensation plan of upgrading to Business Class by default; Extra passengers after upgrading will then automatically accept the over-repayment solution.
- In the application of the infectious disease model, we assume that Beijing and Shanghai conform to the basic premise of the SIR model:

  1. The total population does not have fluctuation, such as birth, death, and mobility, which are not considered, namely $N(t) = K$.

  2. Once a patient comes into contact with susceptible persons, he must have certain infectivity. Suppose at time t, the number of susceptibles that a patient can infect is directly proportional to the total number of susceptibles $s(t)$ in the environment, and the proportionality coefficient is, thus the number of susceptibles that all patients can infect per unit time at time t is $\beta s(t) i(t)$.

3. At time t, the number of patients removed from infected persons per unit time is proportional to the number of patients, with the proportionality coefficient of and the number of patients removed per unit time being $\alpha i(t)$.

Table 1.1: Notations

Notation	Significance
t	day index
T	total pre-sale period
c	maximum capacity of a flight in economy class
n	number of tickets sold already in $[0,t)$ for economy class
ϕ	expected number of tickets to be sold on day t for economy class
k^*	expected ticket demand amount in $[t+1,T]$ for economy class
k	expected number of tickets to be sold in $[t+1,T]$ for economy class
f_t	ticket price on day t of economy class
s	revenue for economy class in $[t+1,T]$
i	number of no-show passengers in economy class
p_N	probability for a passenger to be no-show in economy class
f_N	no-show refund loss for each passenger in economy class
$f_{N'}$	no-show preparation loss for each passenger in economy class
λ_N	no-show refund loss rate in economy class
$\lambda_{N'}$	no-show preparation loss rate in economy class
s_N^k	refund loss caused by no-show passengers on day t in economy class
$s_{N'}^k$	preparation loss caused by no-show passengers on day t in economy class
s_N	total loss caused by no-show passengers in economy class
f_D	bumped compensation for each passenger in economy class
λ_D	bumped compensation rate in economy class
s_D^k	loss caused by bumped passengers on day t in economy class
s_D	total loss caused by bumped passengers in economy class
F_k	total revenue in economy class on day t
c_s	maximum capacity of a flight in business class
n_s	number of tickets sold already in $[0,t)$ for business class
ϕ_s	expected number of tickets to be sold on day t for business class
k_s^*	expected ticket demand amount in $[t+1,T]$ for business class
k_s	expected number of tickets to be sold in $[t+1,T]$ for business class

Table 1.2: Notation

Notation	Significance
f_{t_s}	ticket price on day t of business class
s_s	revenue for business class in $[t+1, T]$
i_s	number of no-show passengers in business class
p_{N_s}	probability for a passenger to be no-show in business class
f_{N_s}	no-show refund for each passenger in business class
$f_{N'_s}$	no-show preparation loss for each passenger in business class
λ_{N_s}	no-show refund rate in business class
$\lambda_{N'_s}$	no-show preparation loss rate in business class
$s_{N_s}^{k_s}$	loss caused by no-show passengers on day t in business class
$s_{N'_s}^{k_s}$	preparation loss caused by no-show passengers on day t in business class
s_{N_s}	total loss caused by no-show passengers in business class
f_{D_s}	bumped compensation for each passenger in business class
λ_{D_s}	bumped compensation rate in business class
$s_{D_s}^{k_s}$	loss caused by bumped passengers on day t in business class
s_{D_s}	total loss caused by bumped passengers in business class
F_{k_s}	total revenue in business class on day t
F	total revenue for the company
HS	heightened security factor
PF	passengers' fear factor

Chapter 2

Literature Review

2.1 Research Status of Overbooking Control [7]

The first overbooking model was proposed by Beckmann, from Tasman Empire airline. The model he proposed used gamma distribution to explain the volume of passengers, and established a mathematical model with the lowest empty seat economic loss and oversold cost, and set a fixed overbooking for each flight. However, the model was not practical because it required the estimation of overbooking cost and passenger demand, and the probability distribution of cancellation of reservation by the reserved passengers [8].

But then, Thompson, who was also from Tasman Empire airlines, proposed a more practical model by completely ignoring the probability distribution of passenger demand and overbooking cost. The model only required a fixed cancellation rate at random, and proposed two important assumptions about the probability of cancellation of a reservation request by a reserved passenger, that is, the probability of a certain reservation cancellation is neither based on whether the passenger belongs to a certain group nor the length of the reservation [9].

Rothstein first introduced the dynamic programming model in the study of hotel revenue management. This model was also applied to solve the problem of airline ticket overbooking

[10-12]. In the 1980s, the research methods of airline ticket overbooking were mostly dynamic methods. Alsaup, a member of SAS (Scandinavian Airlines systems), studied the problem of two-level overbooking through dynamic programming method. Based on the principle of stochastic dynamic programming, P. Zouein and W. Abillama studied the problem of multi flight ticket overbooking. Furthermore, based on the assumption that the passenger demand obeys Poisson distribution, Youyi Feng and Baichun Xiao established a time continuous model and proved the existence of the optimal upper limit of overbooking [13].

In order to consider the impact of airline risk preference on ticket overbooking decision, Jingguang Chen and his team used CVaR (Conditional Value at Risk) to study overbooking decision under different risk tolerance levels [14-16]. Recently, Haotian Zhao and his team carried out relevant research on overbooking decision under different risk tolerance levels by using robust optimization method [17].

2.2 Research Status of Cabin Control [7]

If the random probability distribution of passenger demand does not change, the static cabin control strategy is the optimal decision under the assumption of passenger arrival sequence. However, the probability distribution of passenger demand is uncertain. Therefore, during the whole ticket sales period, airlines constantly update the demand and capacity information, and repeatedly apply static cabin control method is a more conventional method.

The research on the optimization of cabin control shows that if some assumptions in the static research method are relaxed, there will be no optimal decision for static cabin control. Therefore, many scholars use dynamic cabin control to deal with the problems encountered in single segment. Different from static cabin control, the decision-making scheme of dynamic cabin control is not to determine the number of reservation control at the beginning of ticket sales, but to refuse or accept the arrival booking request in real time in the process of absorbing real-time information such as ticket sales and booking requests.

For example, Lee and Hersh regarded the demand of each class of cabin as a non-homogeneous Poisson process, and apply Markov decision-making model to formulate the strategy, that is, to specify the time t. The booking request before t will not affect the strategy at this moment, except for the insufficient transportation capacity. Using discrete-time stochastic process, they proposed a dynamic cabin control problem of single segment [18].

In 2000, Van Slyke and Young, based on Lee and Hersh's model, obtained time continuous optimization results [19], simplified the model to a more specific single segment cabin control problem, and extended it to the non-homogeneous arrival process. The model also recognized the batch arrival of passengers.

2.3 RESEARCH STATUS OF PRICING MODEL [7]

The research focus of airline pricing is from two aspects: one is from the perspective of economics, with the help of relevant theories to analyze the pricing methods and strategies that airlines should adopt, that is to carry out normative qualitative research (theorem driven); the other is to carry out empirical quantitative research by building mathematical models and using real or simulated data (data driven).

As for the effectiveness of revenue management pricing method, Theodore C. Botimer analyzed the efficiency of economics as a foothold. He pointed out that the specified differential pricing system can ensure that passengers can purchase air tickets according to the maximum willingness to spend when supply exceeds demand, so as to optimize the allocation of seats and maximize the income of society and airlines [20].

Chapter 3

The Establishment of dynamic overbooking model

3.1 The Construction of Model

The basic idea for this model is to divide the whole pre-sale period into discrete daily time points. For each day t, we look back to the starting selling day for the total number of tickets sold n in the period $[0, t)$, look forward till the end of the sale period T to predicting the expected selling tickets demand k^* in $[t+1, T]$ and estimate current overbooking amount ϕ [21].

Also, for each flight, we have two classes—-economy class and business class. First, we consider the economy class case.

Suppose that the number of people with the demand of booking plane tickets follows the Poisson distribution, which gives

$$P(X = k^*) = \frac{\lambda^{k^*}}{k^*!} e^{-\lambda} \tag{3.1}$$

where λ is the coefficient in Poisson process.

Also, we can derive the incremental distribution law of Poisson process, which is

$$\begin{aligned} P_{k^*}(t_0, t) &= P(X(t) - X(t_0) = k^*) \\ &= \frac{[\lambda(t - t_0)]^{k^*}}{k^*!} e^{-\lambda(t-t_0)} \end{aligned} \tag{3.2}$$

Therefore, in the future period $[t+1, T]$, the probability for selling k^* tickets is

$$P_{k^*}(t+1, T) = \frac{[\lambda(T - (t+1))]^{k^*}}{k^*!} e^{-\lambda(T-(t+1))} \tag{3.3}$$

In the business class, we can just derive the similar results:

$$P_{k_s^*}(t+1, T) = \frac{[\lambda_s(T - (t+1))]^{k_s^*}}{k_s^*!} e^{-\lambda_s(T-(t+1))} \tag{3.4}$$

3.1.1 The prediction of flight ticket price for each day

For each day, the flight ticket prices can be different. Using statistical data, we can generate a Monte Carlo simulation and determine the economy class ticket price f_t for each day. To be more precise, we divide the prices into multiple price intervals and obtain the accumulated probability for each interval by statistical data. For each day in the pre-sale period, generate a random number between 0 and 1 and rearrange the number array from small to large as they are close to the depart time. For business class ticket price f_{t_s}, the result is similar.

3.1.2 The estimation of actual tickets selling amount

Note that with the influence of flight ticket price fluctuations, the actual number k of people who would order tickets can vary. Thus we introduce a probability function ϵ to show the relation between the ticket price and the fraction of people who will order the tickets among people in demand. In economy class, it is

$$k = \epsilon(f_t)k^* \tag{3.5}$$

By the basic supply-demand model developed in economy, the market demand would decrease as the price of commodity increases [22]. In the airline context, that means the fraction of people who would order tickets is infuenced by ticket prices. There are two basic functions to express the negative correlation: one is inverse proportional function while the other is linear function. Since in the low-price range, there are consumers hesitating for the choices between other transportation ways like trains and they are more sensitive about prices, we apply the inverse proportional function to describe the relation. While in high-price range, consumers tend to choose flights as their first choice and their decisions are less sensitive regarding prices, thus we apply the linear function with negative slope to describe.

$$\epsilon(f_t) = \begin{cases} \dfrac{a}{f_t} & f_t \in (0, B) \\ b f_t + c & f_t \in [B, M] \end{cases} \tag{3.6}$$

where B represents the balance point where two types of consumer's behaviors are separated and M denotes the maximum price the consumer can accept.

In business class, though ticket order amounts would be influenced by the price, the customers are much less sensitive about the price change. Thus we just introduce a simple linear function with negative slope to describe the relation:

$$\epsilon_s(f_{t_s}) = d f_{t_s} + e \tag{3.7}$$

Note that the actual ordering amount k follows the Poisson distribution as well since

$$\begin{aligned} P_k(t+1,T) &= P([X(T)-X(t+1)]\,\epsilon(f_t)=k) \\ &= P(X(T)-X(t+1)=\frac{k}{\epsilon(f_t)}) \\ &= \frac{[\lambda(T-(t+1))]^{\frac{k}{\epsilon(f_t)}}}{(\frac{k}{\epsilon(f_t)})!}e^{-\lambda(T-(t+1))} \end{aligned} \tag{3.8}$$

Also for business class:

$$P_{k_s}(t+1,T) = \frac{[\lambda_s(T-(t+1))]^{\frac{k_s}{\epsilon_s(f_{t_s})}}}{(\frac{k_s}{\epsilon_s(f_{t_s})})!}e^{-\lambda_s(T-(t+1))} \tag{3.9}$$

3.1.3 The dynamic model for overbooking limit [21]

Thus on the day t, the revenue estimated for economy class in $[t+1,T]$ is

$$s = kf_t \tag{3.10}$$

Note that the future ticket prices are estimated by price on current day.

Besides, as the revenue changes with the number of people ordering tickets, it is suggested that the revenue also follows the Poisson distribution as shown below

$$\begin{aligned} P_s(t+1,T) &= P([X(T)-X(t+1)]\,f_t=s) \\ &= P(X(T)-X(t+1)=\frac{s}{f_t}) \\ &= \frac{[\lambda(T-(t+1))]^{\frac{s}{f_t\epsilon(f_t)}}}{(\frac{s}{f_t\epsilon(f_t)})!}e^{-\lambda(T-(t+1))} \end{aligned} \tag{3.11}$$

Therefore, in the period $[t+1,T]$, the expected revenue is

$$\begin{aligned} E(s) &= \sum_{s=0}^{\infty} sP_s(t+1,T) \\ &= \sum_{s=0}^{\infty} s\frac{[\lambda(T-(t+1))]^{\frac{s}{f_t\epsilon(f_t)}}}{(\frac{s}{f_t\epsilon(f_t)})!}e^{-\lambda(T-(t+1))} \\ &= f_t\epsilon(f_t)\sum_{k^*=0}^{\infty} k^*\frac{[\lambda(T-(t+1))]^{k^*}}{k^*!}e^{-\lambda(T-(t+1))} \\ &= f_t\epsilon(f_t)E(k^*) \\ &= f_t\epsilon(f_t)\lambda\,[T-(t+1)] \end{aligned} \tag{3.12}$$

as k follows the incremental Poisson process as shown above.

As for business class, we omit the analogous argument and just give the result here:

$$E(s_s) = f_{t_s}\epsilon(f_{t_s})\lambda_s\,[T-(t+1)] \tag{3.13}$$

For people who have already ordered the tickets successfully, they have two choices: on board in time and no-show (refund the tickets and decide not to take the flight). Thus suppose the probability of a passenger with a ticket to be no-show is p_N. Note that in our model we do not consider the case that passengers travel in group. Thus in the period $[0,t)$ when there are n people ordering the tickets successfully, the distribution of no-show people in economy class is

$$P_i(n) = \binom{n}{i} p_N^i (1-p_N)^{n-i} \tag{3.14}$$

where i is the number of no-show people.

At this stage we can have a brief summary here, as indicated at the beginning of this section, we've determined the number of no-show people in $[0,t)$, the number of expected selling tickets in $[t+1,T]$ and their distributions. We can also determine the overbooking amount ϕ on day t.

Also, the total number of no-show passengers should be computed by all selling tickets, which are the sum of tickets sold n, expected selling tickets k and estimated overbooking amount ϕ.

Note that as the appearance of no-show people is independent of the appearance of selling tickets in the future, we actually have the joint distribution in our model when number of no-show people is j and the number of future selling tickets is k.

$$\begin{aligned} P(X=j,Y=k) &= P_j(n)P_k(t+1,T) \\ &= \binom{n+\phi+k}{j} p_N^j (1-p_N)^{n+\phi+k-j} \frac{[\lambda(T-(t+1))]^{\frac{k}{\epsilon(f_t)}}}{(\frac{k}{\epsilon(f_t)})!} e^{-\lambda(T-(t+1))} \end{aligned} \tag{3.15}$$

Now we can estimate the expected loss caused by no-show passengers and bumped passengers. First, consider the no-show case:

Basically the airline companies are faced with two types of losses for no-show passengers:

(1) Refund for returning tickets

(2) Stuff prepared in advanced for passengers like meals

For customers in economy class, the refund is the product of price and refund rate λ_N, which is

$$f_N = f_t \lambda_N \tag{3.16}$$

Thus we can obtain the refund loss caused by no-show people when there are k tickets to be sold in $[t+1, T]$:

$$s_N^k = i f_t \tag{3.17}$$

The second kind of loss could be computed by the similar method:

$$f_{N'} = f_t \lambda_{N'} \tag{3.18}$$

where $\lambda_{N'}$ is the loss rate for the second type loss.

The loss caused by stuff prepared in advance is given by

$$s_{N'}^{k} = \begin{cases} \left[i - (n+\phi+k-c)\right] f_{N'} & i > n+\phi+k-c \\ 0 & i \le n+\phi+k-c \end{cases} \tag{3.19}$$

as no such loss is caused when the number of no-show passengers is less than the overbooking number. Additionally, losses s_N and $s_{N'}$ follow binomial distribution when they are nonzero as no-show passenger number follows the binomial distribution.

Further, we could get the expected loss $E(s_N)$ and $E(s_{N'})$ when there are k selling tickets in $[t+1, T]$:

$$\begin{aligned} E(s_N^k) &= \sum s_N^k P_i(n) \\ &= \sum_{i=0}^{n+\phi+k} i f_N \binom{n+\phi+k}{i} p_N^i (1-p_N)^{n+\phi+k-i} \end{aligned} \tag{3.20}$$

$$\begin{aligned} E(s_{N'}^k) &= \sum s_{N'}^k P_i(n) \\ &= \sum_{i=n+\phi+k-c+1}^{n+\phi+k} \left[i - (n+\phi+k-c)\right] f_N \binom{n+\phi+k}{i} p_N^i (1-p_N)^{n+\phi+k-i} \end{aligned} \tag{3.21}$$

Similarly, the argument is applied to business class as well, and thus the expected loss in business class is:

$$\begin{aligned} E(s_{N_s}^{k_s}) &= \sum s_{N_s}^{k_s} P_i(n_s) \\ &= \sum_{i_s=0}^{n_s+\phi_s+k_s} i f_{N_s} \binom{n_s+\phi_s+k_s}{i_s} p_{N_s}^{i_s} (1-p_{N_s})^{n_s+\phi_s+k_s-i_s} \end{aligned} \tag{3.22}$$

$$E(s_{N_s'}^{k_s}) = \sum_{i_s=n_s+\phi_s+k_s-c_s+1}^{n_s+\phi_s+k_s} \left[i_s - (n_s+\phi_s+k_s-c_s)\right] f_{N_s} \binom{n_s+\phi_s+k_s}{i_s} p_{N_s}^{i_s} (1-p_{N_s})^{n_s+\phi_s+k_s-i_s} \tag{3.23}$$

Then we derive the results corresponding to bumped passengers.

We may analyze passengers in business class first as it is easier. To compensate business class passengers, the loss value should be

$$f_{D_s} = (1+\lambda_{D_s}) f_{t_s} \tag{3.24}$$

It is obvious that only when the number of no-show passengers is less than the overbooking amount, there would be 'bumped' loss for the company. That is

$$s_{D_s}^k = \begin{cases} 0 & i_s > n_s + \phi_s + k_s - c_s \\ (n_s + \phi_s + k_s - c_s - i_s) f_{D_s} & i_s \le n_s + \phi_s + k_s - c_s \end{cases} \tag{3.25}$$

Note that $s_{D_s}^k$ follows binomial distribution as well for the number of no-show people follows the binomial distribution, and thus the expected loss in bumped case is

$$\begin{aligned} E(s_{D_s}^k) &= \sum s_{D_s}^k P_{i_s}(n_s) \\ &= \sum_{i_s=0}^{n_s+\phi_s+k_s-c_s} \left[n_s + \phi_s + k_s - c_s - i_s \right] f_{D_s} \binom{n_s+\phi_s+k_s}{i_s} p_{N_s}^{i_s} (1-p_{N_s})^{n_s+\phi_s+k_s-i_s} \end{aligned} \tag{3.26}$$

However, for bumped passengers in economy class, there are two options for them:

(1) Upgrade to business class when it has available seats.

(2) Receive compensation f_D

In the first option, there would be no loss for the company while for the second one, the loss value should be

$$f_D = (1+\lambda_D) f_t \tag{3.27}$$

Similarly, the total loss should be

$$s_D^k = \begin{cases} 0 & i > n+\phi+k-c \\ (n+\phi+k-c-i-(i_s-(n_s+\phi_s+k_s-c_s)))f_D & i \le n+\phi+k-c,\ i_s > n_s+\phi_s+k_s-c_s \\ (n+\phi+k-c-i)f_D & i \le n+\phi+k-c,\ i_s \le n_s+\phi_s+k_s-c_s \end{cases} \tag{3.28}$$

When there are available seats in the business class:

$$\begin{aligned} E(s_D^k) &= \sum s_D^k P_i(n) \\ &= \sum_{i=0}^{n+\phi+k-c} \left[n+\phi+k-c-i-(i_s-(n_s+\phi_s+k_s-c_s))\right] f_D \binom{n+\phi+k}{i} p_N^i (1-p_N)^{n+\phi+k-i} \end{aligned} \tag{3.29}$$

When the business class is full:

$$\begin{aligned} E(s_D^k) &= \sum s_D^k P_i(n) \\ &= \sum_{i=0}^{n+\phi+k-c} \left[n+\phi+k-c-i\right] f_D \binom{n+\phi+k}{i} p_N^i (1-p_N)^{n+\phi+k-i} \end{aligned} \tag{3.30}$$

Therefore, in economy class, when n tickets have been already sold, ϕ tickets are expected to be sold on day t and k tickets are expected to be sold in $[t+1, T]$, the total revenue for the airline company should be

$$F_k = (n+\phi+k)f_t - E(s_N^k) - E(s_{N'}^k) - E(s_D^k) \tag{3.31}$$

Observe that F_k is a function of k and so the expected revenue $E(F)$ is

$$\begin{aligned} E(F) &= \sum_{k=0}^{\infty} F_k P_k(t+1,T) \\ &= \sum_{k=0}^{\infty}(n+\phi+k) f_t P_k(t+1,T) - \sum_{k=0}^{\infty} E(s_N^k) P_k(t+1,T) - \sum_{k=0}^{\infty} E(s_{N'}^k) P_k(t+1,T) - \sum_{k=0}^{\infty} E(s_D^k) P_k(t+1,T) \end{aligned} \tag{3.32}$$

The first term on the right hand of the equation is

$$\begin{aligned} \sum_{k=0}^{\infty}(n+\phi+k) f_t P_k(t+1,T) &= \sum_{k=0}^{\infty}(n+\phi) f_t P_k(t+1,T) + \sum_{k=0}^{\infty} k f_t P_k(t+1,T) \\ &= (n+\phi) f_t \sum_{k=0}^{\infty} P_k(t+1,T) + f_t \sum_{k=0}^{\infty} k P_k(t+1,T) \\ &= (n+\phi) f_t + f_t \sum_{k=0}^{\infty} k \frac{[\lambda(T-(t+1))]^{\frac{k}{\epsilon(f_t)}}}{(\frac{k}{\epsilon(f_t)})!} e^{-\lambda(T-(t+1))} \\ &= (n+\phi) f_t + f_t \epsilon(f_t) \sum_{k=0}^{\infty} k^* \frac{[\lambda(T-(t+1))]^{k^*}}{(k^*)!} e^{-\lambda(T-(t+1))} \\ &= (n+\phi) f_t + f_t \epsilon(f_t) \lambda(T+1-t) \\ &= (n+\phi) f_t + E(s) \end{aligned} \tag{3.33}$$

In the whole pre-sale period, the total expected loss is

$$E(s_N) = \sum_{k=0}^{\infty} E(s_N^k) P_k(t+1,T) + \sum_{k=0}^{\infty} E(s_{N'}^k) P_k(t+1,T) \tag{3.34}$$

$$E(s_D) = \sum_{k=0}^{\infty} E(s_D^k) P_k(t+1,T) \tag{3.35}$$

Therefore, the expected total revenue for economy class is

$$E(F) = \sum_{k=0}^{\infty}(n+\phi+k)f_t P_k(t+1,T) - \sum_{k=0}^{\infty}E(s_N^k)P_k(t+1,T) - \sum_{k=0}^{\infty}E(s_{N'}^k)P_k(t+1,T) - \sum_{k=0}^{\infty}E(s_D^k)P_k(t+1,T)$$
$$= (n+\phi)f_t + E(s) - E(s_N) - E(s_D) \tag{3.36}$$

Similarly, the expected total revenue for business class is given by

$$E(F_s) = (n_s+\phi_s)f_{t_s} + E(s_s) - E(s_{N_s}) - E(s_{D_s}) \tag{3.37}$$

and the total revenue for the company is

$$E(F_{total}) = E(F) + E(F_s) \tag{3.38}$$

Note that we cannot figure out the equation above in specific cases as there exists the sum of k from 0 to ∞. Thus we introduce an overbooking limit which is given by

$$U = (1+\lambda_c)c \tag{3.39}$$

where λ_c is the maximum overbooking rate and here we discuss the economy class first.

Thus in the period $[t+1,T]$, when the total selling amount exceeds the overbooking limit, the extra orders would be denied, otherwise they would be accepted, that is

$$P_k = \begin{cases} P_k(t+1,T) & k < U-(n+\phi) \\ 1 - \sum\limits_{k=0}^{U-n+\phi-1} P_k(t+1,T) & k \geq U-(n+\phi) \end{cases} \tag{3.40}$$

Therefore, the expected loss for no-show and bumped passengers are

$$\begin{aligned}
E(s_N) &= \sum_{k=0}^{\infty} E(s_N^k)P_k(t+1,T) + \sum_{k=0}^{\infty} E(s_{N'}^k)P_k(t+1,T) \\
&= \sum_{k=0}^{U-(n+\phi)-1} E(s_N^k)P_k(t+1,T) + E(s_N^{U-(n+\phi)})P_{U-(n+\phi)} \\
&+ \sum_{k=0}^{U-(n+\phi)-1} E(s_{N'}^k)P_k(t+1,T) + E(s_{N'}^{U-(n+\phi)})P_{U-(n+\phi)} \\
&= \sum_{k=0}^{U-(n+\phi)-1} E(s_N^k)P_k(t+1,T) + E(s_N^{U-(n+\phi)})(1-\sum_{k=0}^{U-(n+\phi)-1} P_k(t+1,T)) \\
&+ \sum_{k=0}^{U-(n+\phi)-1} E(s_{N'}^k)P_k(t+1,T) + E(s_{N'}^{U-(n+\phi)})(1-\sum_{k=0}^{U-(n+\phi)-1} P_k(t+1,T))
\end{aligned} \tag{3.41}$$

$$\begin{aligned}
E(s_D) &= \sum_{k=0}^{\infty} E(s_D^k)P_k(t+1,T) \\
&= \sum_{k=0}^{U-(n+\phi)-1} E(s_D^k)P_k(t+1,T) + E(s_D^{U-(n+\phi)})P_{U-(n+\phi)} \\
&= \sum_{k=0}^{U-(n+\phi)-1} E(s_D^k)P_k(t+1,T) + E(s_D^{U-(n+\phi)})(1-\sum_{k=0}^{U-(n+\phi)-1} P_k(t+1,T))
\end{aligned} \tag{3.42}$$

We can derive the similar results for business class, which are

$$\begin{aligned}
E(s_{N_s}) &= \sum_{k_s=0}^{U_s-(n_s+\phi_s)-1} E(s_{N_s}^{k_s})P_{k_s}(t+1,T) + E(s_{N_s}^{U_s-(n_s+\phi_s)})(1-\sum_{k_s=0}^{U_s-(n_s+\phi_s)-1} P_{k_s}(t+1,T)) \\
&+ \sum_{k_s=0}^{U_s-(n_s+\phi_s)-1} E(s_{N_s'}^{k_s})P_{k_s}(t+1,T) + E(s_{N_s'}^{U_s-(n_s+\phi_s)})(1-\sum_{k_s=0}^{U_s-(n_s+\phi_s)-1} P_{k_s}(t+1,T))
\end{aligned} \tag{3.43}$$

$$E(s_{D_s}) = \sum_{k_s=0}^{U_s-(n_s+\phi_s)-1} E(s_{D_s}^{k_s})P_{k_s}(t+1,T) + E(s_{D_s}^{U_s-(n_s+\phi_s)})(1-\sum_{k_s=0}^{U_s-(n_s+\phi_s)-1} P_{k_s}(t+1,T)) \tag{3.44}$$

3.2 Method Adjusted to Covid-19

3.2.1 Discussion of No-Show Probability

Analytic Hierarchy Process (AHP) combines qualitative method with quantitative method. Its central idea is to divide the weight into different levels. The weight setting of factors in each level will directly or indirectly affect the final result. It is a model and method for making decision for complex systems which are difficult to be fully quantified.

Based on the study of relevant literature, the total number of flights and the total number of Covid-19 infection are selected as the evaluation indicators of passenger no-show probability; the degree of passenger fear and the heightened security around the airport are selected as the evaluation indicators, and finally the hierarchical structure as shown in the figure below is established.

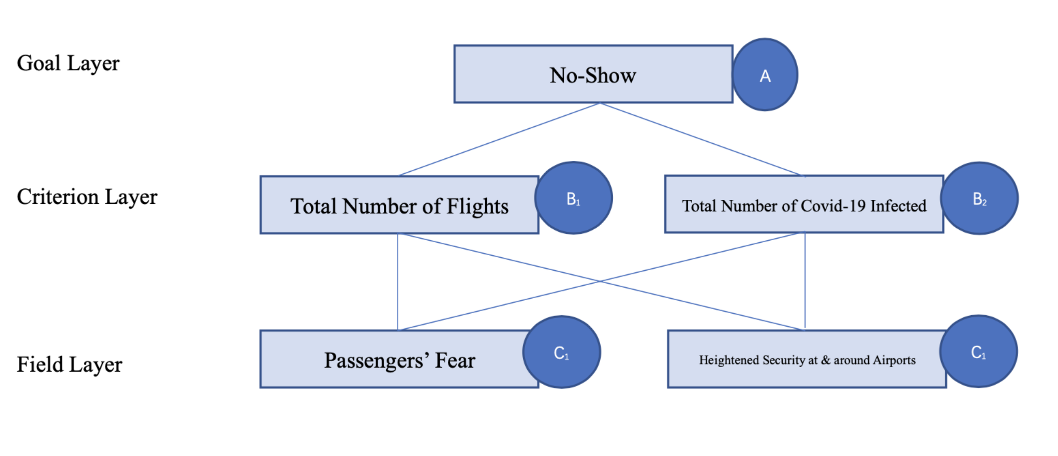

Figure 3.1: Hierarchical Structure of AHP on No-Show

According to "Coronavirus: Airlines say flying is safe, but new study reveals potential for superspreader disaster" (Mercurynews, 2020), it can be known that although the number of airline flights decreases due to Coved-19, passengers are still eager to fly to their destination. Therefore, the pairwise comparison matrix A can be written as

$$A = \begin{bmatrix} 1 & 1/9 \\ 9 & 1 \end{bmatrix}$$

According to the "Coronavirus (COVID-19) Information" (2020) given by U.S. Transportation Security Administration and "Flights Face Delay and Cancellation Amine High Security" (flightglobal, 2006), we can write pairwise comparison matrices B_1 and B_2, as followed,

$$B_1 = \begin{bmatrix} 1 & 1/7 \\ 7 & 1 \end{bmatrix}, \ B_2 = \begin{bmatrix} 1 & 5 \\ 1/5 & 1 \end{bmatrix}$$

Table 3.1: The value of random consistency index RI

n	1	2	3	4	5	6	7	8	9	10	11
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51

From the pairwise comparison matrix A of the criterion layer, the weight vector $w_k^{(2)}$ is calculated and the results are listed in Table 3.2,

Table 3.2: Results of the Criterion Layer

k	1
Maximum Lambda	2
Eigenvector	$\begin{bmatrix} 0.110 & 0.994 \end{bmatrix}^T$
Weight Vector $w_k^{(2)}$ (Normalized Eigenvector)	$\begin{bmatrix} 0.100 & 0.900 \end{bmatrix}^T$
n	2
RI (According to Table 3.1)	0
Consistency Test	Passed

Similarly, from the pairwise comparison matrix Bk of the Field Layer, the weight vector $w_k^{(3)}$ is calculated and the results are listed in Table 3.3.

Table 3.3: Results of Field Layer

k	1	2
Maximum Lambda	2	2
Eigenvector	$\begin{bmatrix}0.141 & 0.990\end{bmatrix}^T$	$\begin{bmatrix}0.981 & 0.196\end{bmatrix}^T$
Weight Vector $w_k^{(2)}$ (Normalized Eigenvector)	$\begin{bmatrix}0.125 & 0.875\end{bmatrix}^T$	
n	2	2
RI (According to Table 3.1)	0	0
Consistency Test	Passed	Passed

Since the random consistency index $RI = 0$ when $n = 2$ (Table 3.1), it is known that both A and B_k can pass the consistency test by $CR = \frac{CI}{RI} < 0.1$.

According to the combination weight vector of the bottom layer (s layer) to the top layer is

$$w^{(s)} = w^{(s)} w^{(s-1)} \cdots w^{(3)} w^{(2)}$$

The final combination weight vector is listed in Table 3.4,

Table 3.4: Combination Weight Vector

k	1	2
Combination weight	0.125	0.833
	0.875	0.167
Combination weight vector	$\begin{bmatrix}0.763 & 0.237\end{bmatrix}^T$	

In order to ensure that the combination weight vector listed in Table 3.4 can be used as the final decision basis, we conduct the combination consistency test. Defined by

$$CI^{(p)} = \left[CI_1^{(p)}, \cdots, CI_n^{(p)}\right] w^{(p-1)}$$

$$RI^{(p)} = \left[RI_1^{(p)}, \cdots, RI_n^{(p)}\right] w^{(p-1)}$$

The definition of combination consistency ratio in layer P,

$$CR^{(p)} = \frac{CI^{(p)}}{RI^{(p)}}, p = 3, 4, \cdots, s$$

And the combination consistency ratio definition of the lowest layer (s layer) to the first layer,

$$CR^{*} = \sum_{p=2}^{s} CR^{(p)}$$

Since all CI are 0 (because $n = 2$), the combination consistency ratio, $CR^{(p)}$, is 0. According to the condition of P level passing the combination consistency test, $CR^{(p)} < 0.1$, only when CR^{*} is appropriately small, the comparison judgment of level A passes the consistency test. Therefore, the combination weight vector listed in Table 3.4 can be used as the basis for the final decision.

Therefore, the proportions of Passengers' Fear PF and Heightened Security HS at and around Airports in passenger no-show probability are 0.763 and 0.237 respectively, according to the combination weight vector listed in Table 3.4.

3.2.2 Quantification Approach

As indicated above, we can express the relation between economy class no-show probability p_N and passengers' fear PF, heightened security HS as

$$p_N = 0.763PF + 0.237HS \tag{3.45}$$

The function is applied to business class as well

$$p_{N_s} = 0.763PF + 0.237HS \tag{3.46}$$

For the passengers' fear factor, we can connect it with actual COVID-19 situation. The idea is, if COVID-19 epidemic situation is serious, passengers would be more concerned about their safety of traveling by plane and the fear factor would matter more, which implies that PF could be larger in quantity. Otherwise we would have a smaller PF.

To better describe the COVID-19 epidemic, Chen et al. applies a time-dependent SIR model. Thus we introduce a simplified SIR model to roughly track the change of COVID-19 infectious population in different time periods and further determine the passengers' fear in quantity PF under different circumstances [4].

First, we discuss in detail about the SIR model. The SIR model we are familiar with is based on the following relationships among the total number of people N, the number of infected people I, the number of susceptible people S, the number of cured people R, and the time T in the epidemic area:

$$\begin{cases} \frac{dS}{dt} = -\beta \frac{IS}{N} \\ \frac{dI}{dt} = \beta \frac{IS}{N} - \alpha I \\ \frac{dR}{dt} = \alpha I \end{cases} \tag{3.47}$$

As we've discussed in class, the infectious population basically has three stages:

(1) Increase slowly

(2) Increase dramatically till the turning point

(3) Decrease

Therefore, it is obvious that the shape of infectious population curve is like a "hat". Thus we could use it to divide time periods and for each period, we could set a constant to characterize to what extent passengers are concerned about their safety. The criteria to divide these three intervals is:

(1) Low fear: the time intervals when the infectious population is less than $p\%$ of the population sum of city A and city B.

(2) Medium fear: the time intervals when the infectious population is between p% and q% of the population sum of city A and city B.

(3) High fear: the time intervals when the infectious population is between q% of the population sum of city A and city B and the highest infectious population.

For low fear, we set $PF = 0.1$ and make $PF = 0.3, 0.5$, respectively for medium fear, high fear cases.

As for heightened security HS, we basically use the time for security check to quantify this factor. Normally passengers reach 2 or less hours before departure. Due to the impact of the COVID-19, it would take them one or more hours to assist epidemic prevention inspection. Thus for passengers if they spend one more hours than before, starting from 0, HF factor would increase by 0.2 until it reaches 0.6.

Chapter 4

Application to Real Practices

4.1 Data Analysis

With the constructed model we have previously deduced, we then apply the method to real practices, with the assistance of MATLAB 2020a.

Here we take Air China's Flight CA1831 from Beijing to Shanghai as the main source of data and discuss in two cases:Business Class(BC) and Economic Class(EC). By reference to the data shown on the official website and relative calculation, we can obtain the the intensity of Poisson is 2.38 for passengers in Business Class and 11.25 for passengers in Economy Class.We have 30 available seats and the maximum of overselling tickets by regulation are 50 for BC, 120 available seats and the maximum of overselling tickets by regulation are 180 for EC. Considering the real life cases, the passengers in BC has a higher probability of not-showing up, while they can get all the money back; the passengers in EC has lower probability of not-showing up, and they can only get 67% of the total ticket price for this action. Similarly, the reparation for DB passengers in BC is much higher than those in EC, which we adopt the average value here, namely 2.72 and 1.23 separately.

4.1.1 Plane Pricing Method

Airlines generally adopt a reservation policy for tickets. Customers can make reservations by phone or over the Internet, which is highly uncertain and is likely to be canceled for a variety of reasons. In order to maximize profits, airlines need to win customers on the one hand and reduce the loss caused by customers' cancellation of reservation on the other hand. To manage this, the Airline company adjusts the plane price along with time according to market demand. Specifically, ticket price in the peak season tends to be higher than in the off-season. Besides, prices are lower at the beginning of the sale and higher near the departure time. Therefore, we speculated here, that the air ticket price of a plane in the same airline would show a periodic fluctuation according to the time series, and thus, we can apply the Fourier series first to fit the price from the beginning of ticket sale to the departure of the plane, so as to provide a basis for the next step of profit calculation.

Take Air China's Flight CA1831 from Beijing to Shanghai as an example. We take the average daily price of first-class and economy class from April 2019 to February 2020 and use the Fourier function to simulate, and the results are shown in the figure below.

The statistic values turn out to be: R-square=0.2111, DFE=249, Adj R-square=0.1699, RMSE=381.9890, which illustrate that the periodic simulation does not fit the data well. The Plane Price is not periodic perfectly under the influences of many complicated circumstances.

We speculate that the price of the aircraft is affected by complex factors such as holidays and weather, which does not correspond to the cyclical fluctuations we have predicted. Therefore, we use the Monte Carlo simulation method to calculate the corresponding probability of air ticket prices in the past year. According to the principle that the closer the departure time is, the higher the air ticket price is, the predicted air ticket prices are then sorted in ascending order as the predicted prices in 30 days separately before the departure time.

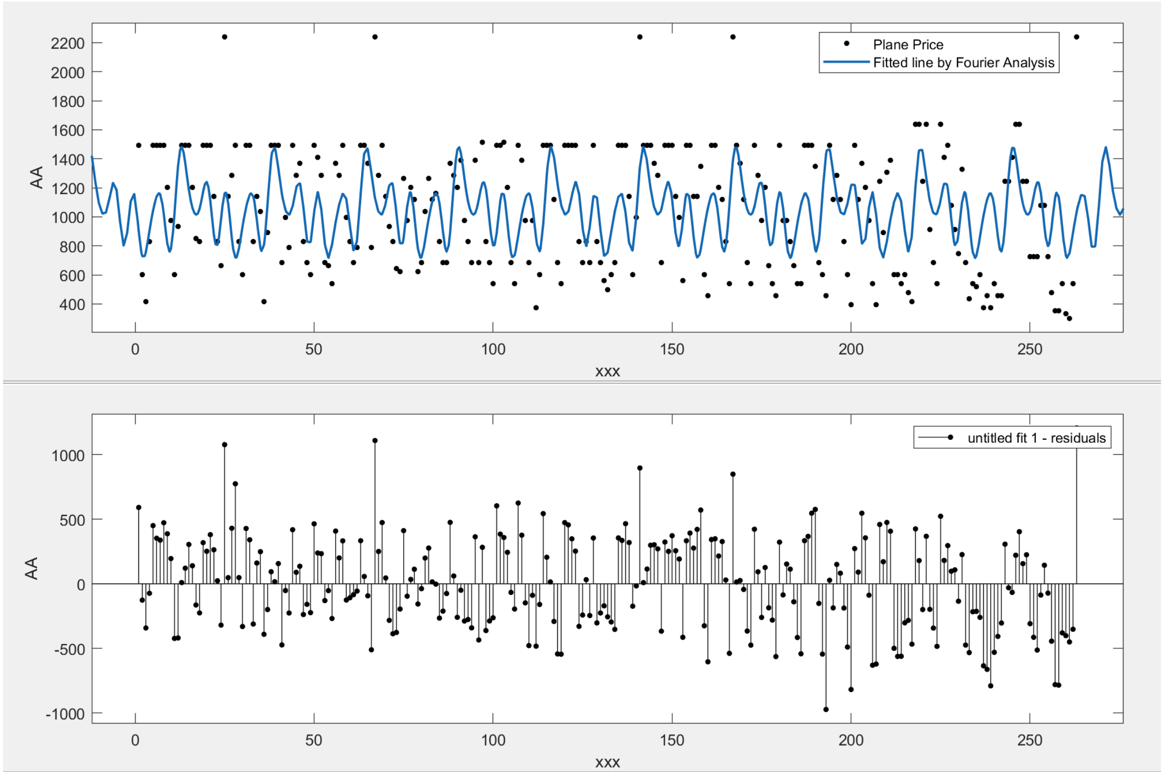

Figure 4.1: The Fourier Simulation of Plane Price

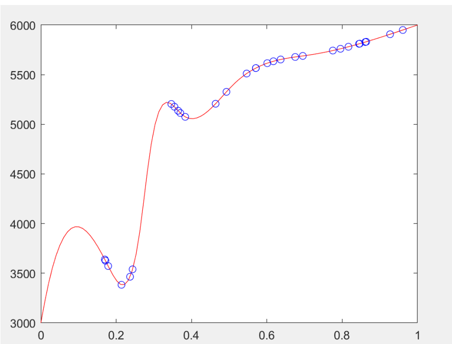

Figure 4.2: The Fourier Simulation for Business Class

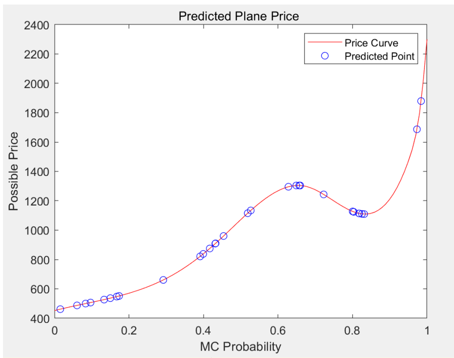

Figure 4.3: The Fourier Simulation for Economy Class

4.1.2 Demand Curve Prediction

According to the classical Supply- Demand curve in economics, the change in price and the change in quantity demanded are inversely proportional. Other conditions being equal, this relationship between price and demand is called the Price-Demand curve. The demand table and demand curve below reflect the relationship between product price and demand quantity, that is, they vividly describe the relationship between price and demand in the demand function. However, adding other factors into consideration such as time would cause these conditions to change correspondingly, and the shape and position of the demand curve will change, as will passenger demand for airlines as a whole.

We note that the demand for business class tickets does not fluctuate significantly with the increase in ticket prices due to the large budget of business class passengers. With reference to previous data, their demand tends to have a linear relationship with the plane price. However, passengers in economy class, with relative low budgets, are significantly affected by the price. Here, we according to the actual need for the inverse proportion of supply and demand curves is improved, access to the high-speed rail tickets from Beijing to Shanghai is about 400 yuan, so we are in 400 as a turning point, in 400 after the curve into a linear function, in the highest acceptable price economy class passengers (here we forecast of economy history highest) fellowship with x weeks. By referring to the relevant values and making an estimate, we can get the function of the ticket change of the ticket demand of business class passengers and economy class passengers respectively, so as to simulate the actual number of ticket buyers per day based on the ticket prices we have predicted in the previous step.

4.1.3 Daily Sales of Tickets

As discussed in the previous section, the passengers with daily booking requirements are in line with the Poisson distribution. Therefore, the Poisson distribution is used to simulate

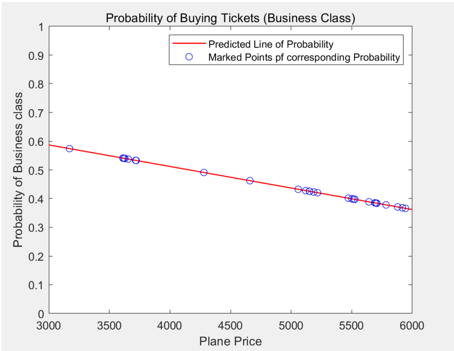

Figure 4.4: Prediction Probability for Business Class

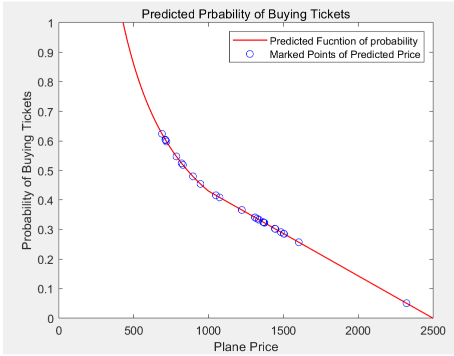

Figure 4.5: Prediction Probability for Economic Class

the number of passengers with booking requirements each day in the 30 days before the plane takes off, and then the obtained result is multiplied by the booking probability of the previous simulation respectively to obtain the actual number of passengers per day.

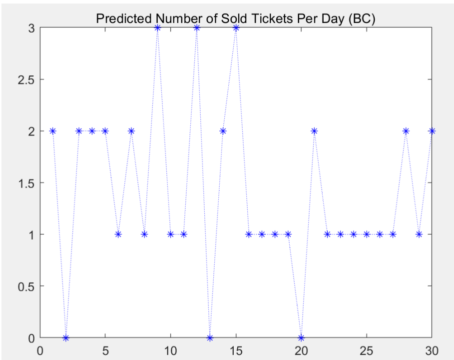

Figure 4.6: Prediction Probability for Business Class

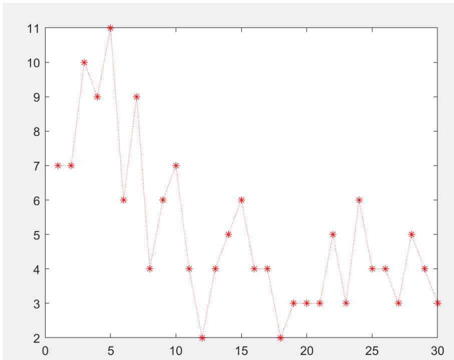

Figure 4.7: Prediction Probability for Economic Class

Similarly, it was discussed in the previous chapter that the no-show probability of each passenger booking conforms to the quadratic term distribution.Therefore, we can make profit estimates for any day in the 30 days based on the daily number of tickets fitted, and determine the best expected sales tickets for that day by selecting the most profitable combination among all possible results.If the actual number of sold tickets is less than the best-predicted number of sold tickets , we shall accept all tickets; If the actual number of sold tickets is greater than the best-predicted number of sold tickets, we choose to reject the extra orders that exceed the best-predicted number.

Take one run when t=20 (which refers to the day that is 20 days before the departure date) as an example, by calculating the total profit and final profit considering passengers' No-show probability, we can get the relationship between predicted profit and amount of sold tickets.

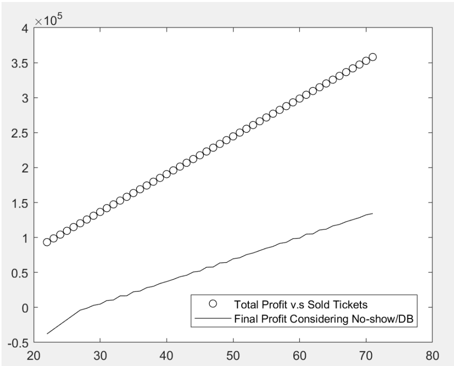

Figure 4.8: Total & Final Profit (BC) When t=20

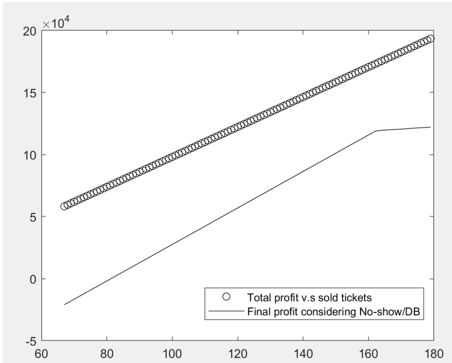

Figure 4.9: Total & Final Profit (EC) When t=20

Thus, we can get from the picture that, overall, the total profit has a linear relationship with the sold tickets, but there is a turning point for Economic Class– When there are more sold tickets, airline companies have to pay for DB and thus leads to the slightly decreasing rate of the increase of the profit, which should be paid attention to.

Running the method, the result we obtain is that the best-predicted selling number for Business Class is 0 while our predicted number of buying demand is 1, thus, we reject one order for Business Class; for Economic Class, we have the best predicted number as 25 and the real buying demand is 8, thus, we accept all the orders.

For one-time simulation, by calculating for all t values, we can get the result as below:

The total profit followed by our constructed dynamic model is 1.996×10^5 Yuan, which is 11% higher than the profit of 1.786×10^5 generated by the static model that the airline company currently has. That indicates the feasibility of our model.

	One-run Experiment for Dynamic Model							
	Best-predicted	Simulation	Successful Orders	Rejected Orders	Best-predicted	Simulation	Successful Orders	Rejected Orders
1	3	1	1	0	7	9	7	2
2	2	1	0	1	11	10	10	0
3	3	2	0	0	8	7	7	0
4	4	2	0	0	7	5	5	0
5	2	1	1	0	8	9	8	1
6	2	2	0	0	7	5	5	0
7	2	2	1	0	7	7	7	0
8	3	2	1	0	9	8	8	0
9	1	1	2	0	6	4	4	0
10	2	1	1	0	8	5	5	0
11	2	1	1	0	10	5	5	0
12	1	2	1	1	7	6	6	0
13	3	2	2	0	8	6	6	0
14	3	1	1	0	6	5	5	0
15	2	1	1	0	6	6	6	0
16	3	2	2	0	6	5	5	0
17	3	2	2	0	7	5	5	0
18	2	1	1	0	6	6	6	0
19	2	1	1	0	7	4	4	0
20	2	2	2	0	5	4	4	0
21	3	1	1	0	6	5	5	0
22	2	1	1	0	6	3	3	0
23	2	2	2	0	5	3	3	0
24	3	1	1	0	5	5	5	0
25	2	1	1	0	4	3	3	0
26	4	2	2	0	7	6	6	0
27	1	0	0	0	6	5	5	0
28	1	2	1	0	5	4	4	0
29	2	1	1	0	5	3	3	0
30	3	2	2	0	7	3	5	0

Figure 4.10: The Fourier Simulation of Plane Price

4.2 Code adjusted to Covid-19

4.2.1 Influences on Plane Price

As we mentioned in background information, due to the outbreak of coVID-19, traffic restrictions have been implemented in various regions, resulting in a significant decrease in passengers and a rise in the vacancy rate of airlines, which has forced them to adopt corresponding policies.Here, we extract the daily number of newly infected people in China and the corresponding air ticket price of CA1831 on that day, and use MATLAB simulation image to conclude that the two have an obvious inverse relationship.

The statistic value of this fitted curve is: $SSE = 3.092\times10^6, R-square = 0.4675, AdjustedR-squareis 0.4562$ and $RMSE = 256.5$, which shows a relatively obvious significance of the fitted line. That indicates that the Covid-19 does have significant negative influence on air transportation. People's not showing up and cancelling orders due to the spread disease push the plane price down, since the decreasing market demand forces the market price to

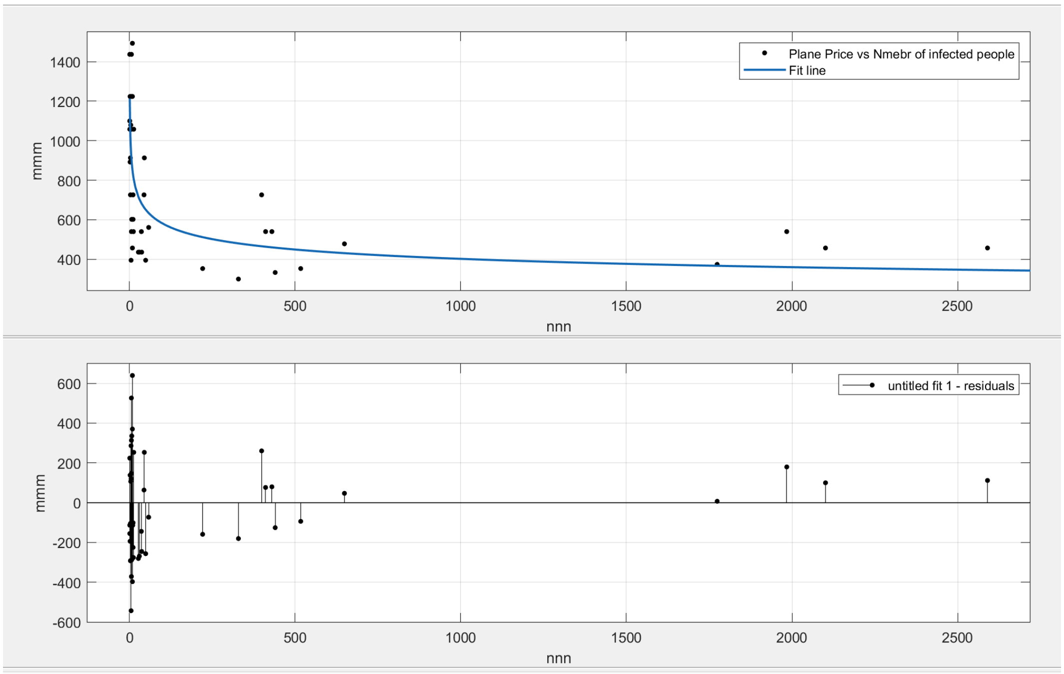

Figure 4.11: The Estimated SIR Model of Covid-19

decrease as well.

The result also verifies our discussion in the previous chapter: the rapidly increasing number of infected people gives rise to heightened security and adds fires to passengers' fear, which will increase the probability of showing up. To quantify the security and fear, we adopt the hours that passengers have to spend going through the security (they need to scan the health code, or check the body temperature) and the total number of infected people in the region separately.

4.2.2 SIR model for COVID-19

As indicated above, SIR model can be applied to track the change of infectious population.

Early at the beginning of the spread, the total number of vulnerable groups is the total number of people, namely $S = N$, then we can simplify the infections, based on the relationship between vulnerable group I and time t is:

$$\frac{dI}{dt} = \beta\frac{S}{N}I - \alpha I = (\beta - \alpha I)$$

Thus, we can obtain the solution to this equation:

$$I(t) = e^{(\beta-\alpha)t}$$

This relationship indicates that the approximate total number of infected persons is an exponential function of time. The constant β and α should be determined according to the characteristics of the outbreak, so that we can realize the estimation of infections. Meanwhile, the epidemic prevention and control measures also affect these parameters and in turn reflect the effectiveness of prevention and control measures. These parameters are generally based on epidemiological statistics and will be reflected in the course of the epidemic. That is, we can also determine these parameters based on actual outbreak reports.

Since we have accumulated some real epidemic data, retrospective fitting based on SIR analysis can accurately determine these parameters.

First, using the CTFtools in MATLAB to do the least-square simulation of equation (4.1) based on early public data yielded initial parameter estimates of $\beta = 0.0000003$ and $\alpha = 0.0077266$. Using these parameters and equation, we found that the actual number of early infections and estimates were very consistent.

From the obtained data we can see that the model can fit the number of infected persons very well at the beginning of the spread process, since after a set of days there will be much more external factors forcing on the real number, such as the government's newly implemented policies or the increasing power of people's immune system and their corresponding precaution strategies. Thus the specific values of α and β varies along with time. Here for convenience, we just use the result we get by simulating the early stage of disease

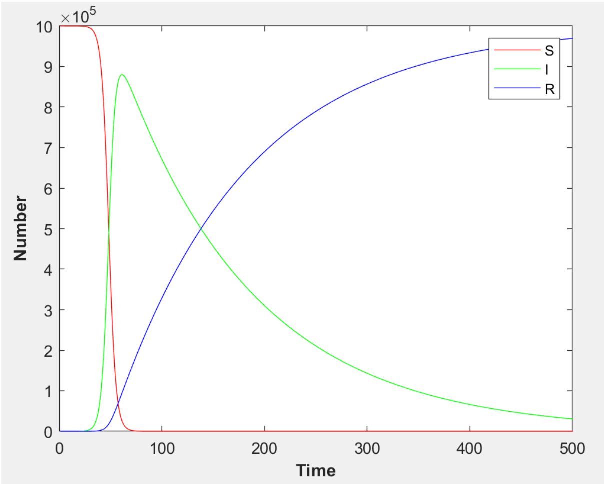

Figure 4.12: The Estimated SIR Model of Covid-19

spread to predict the whole process (Taking the assumption that Beijing and Shanghai are isolated islands) and classify the corresponding number into different level set to give a vivid illustration of passengers' fear.

Following the strategies we have introduced in the previous chapter, we can get the corresponding p_n, namely the probability of passengers' not showing up, for each period. Since we are in the fourth period now, we then run out the dynamic model and generate the expectation of air companies' profit which is shown below. From this we can get that the final profit has a positive linear relationship with the expected number of ticket orders.

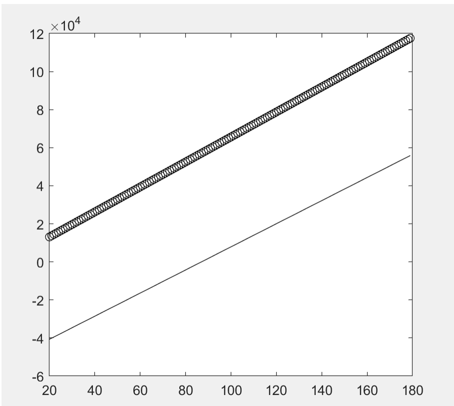

Figure 4.13: Total & Final Profit (BC) When t=20

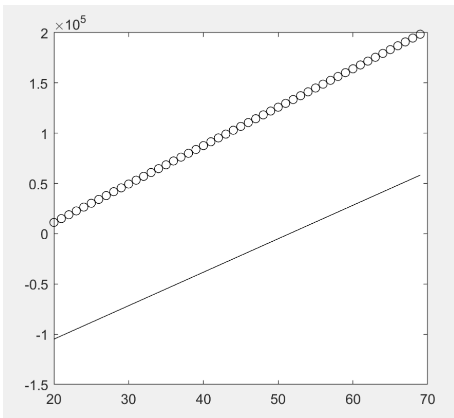

Figure 4.14: Total & Final Profit (EC) When t=20

That indicates that, even the airplane reaches the maximum limit of ticket orders set by the government regulations, the expectation of showing-up passengers can not fulfill the total seats, thus leads to the linear relationship of total profits. Thus, to boost the profit, here we suggest the airline companies should increase the maximum limit of ticket orders to avoid loss of the potential benefits. We can verify the model by generating the data set of the result of our dynamic model as well.

One-run Experiment for Dynamic Model Adjusted to Covid-19

	Empty Seats	Best-predicted	Simulation	Successful Orders	Rejected Orders	Av	Empty Seats	Best-predicted	Simulation	Successful Orders	Rejected Orders
1	4	7	1	1	2	4	0	44	9	9	0
2	4	9	2	2	0	4	0	37	3	3	9
3	3	11	3	3	0	3	0	39	6	6	0
4	2	14	3	3	0	2	0	40	8	8	0
5	0	16	1	2	0	0	0	36	8	8	0
6	0	17	2	2	0	0	0	36	8	8	0
7	0	16	1	1	0	0	0	36	8	8	9
8	0	19	3	3	0	0	0	35	7	7	0
9	0	17	1	1	0	0	0	35	7	7	0

Figure 4.15: The Dynamic Model Running under Covid-19

Therefore, we can conclude that due to the impact of COVID-19, the no-show probability of passengers has been significantly increased. Even if the airline sells the maximum number of advance sale tickets stipulated in its policy, the passengers finally cannot make the plane full, which also confirms the phenomenon of increasing the vacancy rate of flights in reality.

In the fitted dynamic model equation shown above, we can see that the corresponding optimal predicted tickets per day are significantly increased,indicating that to obtain the maximum profit, the most effective measure that airlines should take is, to increase the maximum pre-sale tickets in the airline regulations, such that they can make the aircraft as full as possible. However, considering personal safety under the influence of COVID-19, the demand of passengers stays low and will not increase correspondingly to the increase of maximum. Thus, we can obtain the converse direction, airline companies should reduce the size of the aircraft to boost the efficiency of transportation.

Besides, notice that when passengers' no-show possibility increases, the result of our dynamic model tends to be consistent with the static model that the airplane has adopted. That shed lights on the application range of our model– it performs better when passengers' no-show possibility remains relatively low, or the demand for the tickets turns out to be relatively high, that means, when there is a "need" to decide whether we should oversell, instead of this case, that the passengers will not fulfill the plane with high no-show probability and low demand for travelling. In a word, the model is used to decide "how to oversell" rather than "whether to oversell".

Thus, we then assume that the airline companies have adopted the airplane with a smaller size, with the capacity of 80 for EC and 20 for BC (Here we assume that the airline company has replaced CA1831 with a new one, other variables being same). By running MATLAB, we can get the following optimal result:

			One-run Experiment for Dynamic Model Adjusted to Covid-19					
	Best-predicted	Simulation	Successful Orders	Rejected Orders	Best-predicted	Simulation	Successful Orde	Rejected Orders
1	6	1	1	0	9	12	9	3
2	6	2	2	0	9	8	8	0
3	5	1	1	0	7	6	6	0
4	4	2	2	0	8	6	6	0
5	4	2	2	0	8	10	10	2
6	4	2	2	0	11	9	9	0
7	5	1	1	0	10	8	8	9
8	6	0	0	0	7	8	8	1
9	6	2	2	0	9	9	9	0
10	3	3	1	0	7	6	6	0
11	3	1	1	0	9	11	11	2
12	3	1	1	0	10	6	6	0
13	3	2	2	0	9	8	8	0
14	4	2	2	0	8	5	5	0
15	4	2	2	0	6	4	4	0
16	3	2	2	0	6	4	4	0
17	3	1	1	0	7	4	4	0
18	5	0	0	0	5	2	2	0
19	5	2	2	0	5	5	5	0
20	3	2	2	0	7	4	4	0
21	3	2	2	0	6	4	4	0
22	4	0	0	0	6	5	5	0
23	5	2	2	0	5	3	3	0
24	4	1	1	0	6	4	4	0
25	4	1	1	0	7	4	4	0
26	5	1	1	0	7	4	4	0
27	4	2	2	0	6	4	4	0
28	4	2	2	0	5	3	3	0
29	4	1	1	0	4	3	3	0
30	5	2	2	0	3	4	4	1

Figure 4.16: The Dynamic Model Running under Covid-19 for Smaller Aircraft

With reference to the previous method we have stated before, we calculated the total revenue for the outcome of our dynamic model and the static model, discovering that the total profit is approximately 7% higher than the static one. Thus, we can get to the conclusion that, our dynamic model does works for different situations with various parameters, both under normal condition and COVID-19 condition, when there needs the decision on "overselling" strategies.

Chapter 5

Discussion & Conclusion

5.1 Conclusion

In conclusion, we generate a dynamic model which offers the suggested selling number of tickets for airline companies considering the overbooking strategy. Combining the Monto-Carlo simulation for plane price and the basic Price-Demand Curve in Economics, we give a prediction of ticket orders for each day and generate the optimal result that maximizes the companies' profit. Under normal situations, our dynamic model will generate 11% higher profit than the static model which does not consider the overbooking amount on each day. In the real context, the airline could follow the model to determine the tickets selling strategy on each day in the pre-sale period.

The constructed dynamic model decides "how to oversell" instead of "whether to oversell". That explains its similar performance to the static model when considering COVID-19 epidemic with an increase in passengers' no-show probability. Further, the result also shed lights on a possible strategy would be to use smaller planes to carry flight tasks to make more orders than the plane's capacity, since increasing the maximum overselling ticket amount will not better the situation. In that case, we obtain that the dynamic model would perform better than the static model as well with approximately 7% increase in total profit. That verifies the validity in different situations of the dynamic model, which will have certain significance of reference for the overbooking strategy adopted by airlines in the future whether under normal or unnatural conditions.

5.2 Discussion

5.2.1 Advantages

Dynamic Overbooking Model:

- Compared with static model which only provides an overbooking limit without detailed strategy for ticket sales on each day, this model instead gives thorough sales strategy with total ticket selling amount and prices on each day to maximize the revenue.
- To better describe the real ticket price fluctuations, the model applies statistical data to predict price changes to better determine the ticket selling strategy.
- The model considers consumers' behaviors as well. Using basic demand curve in economy principle, it can reflect real market demand changes with respect to ticket price changes.

AHP Model:

- It is a systematic analysis method. It does not cut off the influence of each factor on the result. The weight setting of each layer in AHP will directly or indirectly affect the result, and the influence degree of each factor in each level on the result is quantitative and clear. This method can be used especially for the system evaluation without structural characteristics and multi-objective, multi criteria, multi period and so on.

- The combination of qualitative method and quantitative method makes the complex system decompose. It can make people's thinking process mathematically and systematized, which is easy for people to accept.

- Less quantitative data is needed. AHP is mainly based on the evaluator's understanding of the nature and elements of the evaluation problem, which is more demanding than the general quantitative method.

5.2.2 Disadvantages

Dynamic Overbooking Model:

- Due to the limitation of our computation capacity, we could not simulate multiple flights in different routes for an airline company at the same time.

- Currently on each day, we use the ticket price on that day to compute the revenue expectation after that day. While in real case, ticket prices tend to be higher when the flight is close to departure. However, as we use overbooking amount as the dynamic parameter in our model, it would be much harder to consider prices as dynamic.

AHP Model:

- It can not provide new solutions for decision-making. The function of AHP is to select the better one from the alternatives, so it will be fixed by the alternatives.

- Less quantitative data and more qualitative components make it difficult to be convincing.

5.2.3 Improvement directions:

Dynamic Overbooking Model:

- To better compare with normal situations, we use the same Poisson process parameter λ, λ_s to describe the demands of passengers in COVID-19 case. However, in the real settings, COVID-19 would decrease the passengers' demands to travel by plane. Thus an individual Poisson process parameter could be applied to characterize the COVID-19 airline situations better.
- Under COVID-19 situation, currently it is a little arbitrary to determine the maximum overbooking limit for airline companies to maximize the revenue. We could construct a relation between no-show probability and maximum overbooking rate on each day.

AHP Model:

- Find the method of quantitative reference value or look for quantitative statistical data.
- More accurate screening of the parameters involved, discarding the parameters with less relevance to the target. Consider other potential target related parameters.

SIR Model:

In the chapter on Application, we use a simple SIR model to simulate the infection situation in Beijing and Shanghai. Thus, we automatically assumed that Beijing and Shanghai meet the basic conditions of the SIR model, that is, both of them are "isolated islands" without any interference from external factors. Therefore, the values of and are applicable in the early stage of virus transmission, but in the middle and late stages of virus transmission, the SIR model based on this parameter cannot accurately predict the actual number of infections due to the interference of other external factors. Therefore, in the process of improving the model, we can consider more variables, such as the prevalence of secondary infections, or adjust the value of and in real-time, which makes the actual prediction more accurate.

List of Figures

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